Approximately Optimal Surveillance Test Policies for Two-Unit Parallel Standby System

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Introduction

- Safety systems and standby systems
- Availability vs. Reliability
- Safety vs. cost
Single-Unit Systems

- Unavailability Models: an approximate optimal test interval to minimize the system unavailability is given by

$$STI_u = \sqrt{\frac{2\tau}{\lambda}},$$

where

- $\tau = \text{expected downtime per cycle for testing, maintenance and repair}$
- $\lambda = \text{component failure rate}$

Examples: Apostolakis and Chu (1980), Vaurio (1979)

- Combined Models: the approximate optimal test interval balancing system unavailability and maintenance cost
Multi-Unit Systems

- Vaurio and Sciaudone (1979): analytical expression for average system unavailability of redundant systems; optimal test interval obtained by a computer code

- Apostolakis and Chu (1980): uniform staggered testing minimizing system unavailability for two identical units in parallel

- Lee et al. (1990): analytical expression for average unavailability of 1-out-of-2 systems with (almost) simultaneous testing; optimal test interval obtained numerically
Multi-Unit Systems

- Uryas’ev and Vallerga (1993): method for obtaining optimal test intervals for general standby systems, but ignoring time-dependent component unavailability

- Vaurio (1995): “For systems with multiple components it is not generally possible to solve optimal test intervals in analytical form, even if only the system unavailability is optimized without cost consideration”
Model Features

- **Objective Function**: considering both maintenance cost and loss due to system unavailability

- **System Unavailability**: assessed taking into account time-dependent component unavailability

- **Decision Variables**: including both test interval and testing scheme for multi-unit systems (not limited to pre-specified alternatives)
Assumptions

- Units are either operable or failed.
- Failure modes include both random failure during standby, and failure on demand.
- Following testing, restorative maintenance is performed if the unit was operable. Otherwise, the unit is fully repaired or replaced.
- Units are unavailable during testing.
- Units are as good as new after restoration.
Assumptions

- Surveillance tests of unit $k$ are performed with a fixed period $STI_k$.
- If the units have unequal test intervals, the test interval of the system, $STI$, is equal to the longest of the unit test intervals, and is an integer multiple of each unit test interval.
- Exactly simultaneous testing is not modeled.
Maintenance Schedule for Two-Unit Systems

\[ STI = \text{surveillance test interval} \]
\[ L_{12} = \text{time lag between tests of units 1 and 2} \]
\[ O_i = \text{start of } i^{th} \text{ test} \]
\[ \tau_k = \text{test duration of unit } k \]
\[ A_i = O_i + \tau = \text{completion of } i^{th} \text{ test} \]
Objective Function

\[ M(\text{STI}, L) = C_u \bar{U}(\text{STI}, L) + \frac{1}{\text{STI}} \sum_{k=1}^{K} N_k [C_{T_k} + C_{F_k} u_k (\text{STI}_k - \tau_k)] \]

where

- \( C_u = \) expected cost per time unit of system unavailability
- \( \bar{U}(\text{STI}) = \frac{\int_0^{\text{STI}} u(t) \, dt}{\text{STI}} = \) average system unavailability per cycle
- \( C_{T_k} = \) test cost of unit \( k \) (per test)
- \( C_{F_k} = \) additional cost for repair unit \( k \) if failure is detected
Objective Function

\[ M(STI, L) = C_u \tilde{U}(STI, L) + \frac{1}{STI} \sum_{k=1}^{K} N_k [C_{T_k} + C_{F_k} u_k(STI_k - \tau_k)] \]

where

- \( STI_k \) = test interval of unit \( k \)
- \( STI = \) length of cycle = \( \max\{STI_k\} \)
- \( N_k = \) number of tests of unit \( k \) in each cycle
- \( L = L_{12} = \) time lag between tests of units 1 and 2
Unit Unavailability (time-dependent)

\[ u_k(t) = \begin{cases} 
1 & \text{in testing period} \\
\rho_k + (1 - \rho_k)F_k[x_k(t)] & \text{in standby period}
\end{cases} \]

where

\[ x_k(t) = \text{age of unit } k \text{ at time } t \]
\[ \rho_k = \text{failure probability on demand of unit } k \text{ (possibly including maintenance-induced failure if appropriate)} \]
\[ F_k(x_k) = \text{probability that unit } k \text{ fails randomly during standby, at or before age } x_k \]
Step I: Finding the optimal test interval

For any given $STI$, find the optimal testing strategy by solving the following problem:

$$\min_{L} M(STI, L)$$

subject to

$$L_{jk} \geq \tau_j$$
$$STI \geq \sum L_{jk} + \tau(K)$$
Step II: Finding the optimal testing strategy

For the optimal testing strategy, find the optimal test interval $STI^*$ such that

$$\frac{d}{dSTI} M_L(STI)|_{STI^*} = 0$$

$$\frac{d^2}{dSTI^2} M_L(STI)|_{STI^*} > 0$$

and $STI^* \geq \sum_k \tau_k$
Two Identical Units with Independent Failures

- $L^* = \frac{1}{2} STI$, i.e., uniformly staggered testing was found to be the best strategy for two identical units in parallel. This result holds even if common-cause failure of both units is considered.
- An analytic expression of the approximately optimal test interval was obtained using a first-degree Taylor series polynomial.
Two Non-identical Units with Independent Failures

The optimal time lag is given by

\[ L_{12} = \frac{1}{2} STI + \Delta L, \]  

(1)

where

\[ \Delta L = \begin{cases} 
\frac{1}{2} STI - \tau_2 & \text{(almost simultaneous testing, unit 2 tested first)} \\
\frac{1}{2} \left( \frac{1}{2} (\tau_1^2 - \tau_2^2) + \frac{\tau_1}{\lambda_1} - \frac{\tau_2}{\lambda_2} \right) & \text{if} \ \frac{1}{STI} \left[ \frac{1}{2} (\tau_1^2 - \tau_2^2) + \frac{\tau_1}{\lambda_1} - \frac{\tau_2}{\lambda_2} \right] \geq \frac{1}{2} STI - \tau_2 \\
-\frac{1}{2} STI + \tau_1 & \text{(almost simultaneous testing, unit 1 tested first)} \\
\frac{1}{STI} \left[ \frac{1}{2} (\tau_1^2 - \tau_2^2) + \frac{\tau_1}{\lambda_1} - \frac{\tau_2}{\lambda_2} \right] & \text{if} \ -\frac{1}{2} STI + \tau_1 < \frac{1}{STI} \left[ \frac{1}{2} (\tau_1^2 - \tau_2^2) + \frac{\tau_1}{\lambda_1} - \frac{\tau_2}{\lambda_2} \right] < \frac{1}{2} STI - \tau_2 \\
\frac{1}{STI} \left[ \frac{1}{2} (\tau_1^2 - \tau_2^2) + \frac{\tau_1}{\lambda_1} - \frac{\tau_2}{\lambda_2} \right] & \text{(staggered testing)} \\
\end{cases} \]
Sensitivity Analysis—Equal Test Interval

Objective Function Value

Approximately Optimal Test Interval and Time Lag

Observations:

1. When $C_u$ is small ($STI_\lambda$ large), the optimal testing strategy is staggered (not necessarily uniformly), but the impact of the testing strategy is very small.

2. When $C_u$ is large ($STI_\lambda$ small), almost simultaneous testing is very close to the optimal test strategy.
Sensitivity Analysis-Unequal vs. Equal Test Intervals

**Observation:** Unequal test intervals give better objective function values than equal test intervals.
Approximately Optimal Surveillance Test Policies for Two-Unit Parallel Standby System

- Systems
- Two Identical Unit, with CCF on demand

\[
P\{\text{system fails to start due to CCF}\} = \beta\rho \\
P\{\text{systems fails to start not due to CCF}\} = [(1 - \beta)\rho]^2 \\
P\{\text{only one unit fails on demand}\} = 2\{\rho - [\beta\rho + (1 - \beta)^2 \rho^2]\} \\
P\{\text{no demand failure}\} = 1 - 2\rho + \beta\rho + (1 - \beta)^2 \rho^2
\]
Sensitivity Analysis—Less Important System with Small $\rho$

**Average Cost**

**Approximately Optimal Test Interval**

**Observations:**

1. The optimal average cost increase with the dependence ($\beta$) between units.
2. The optimal test intervals are not affected by the dependence between units.
Sensitivity Analysis—Less Important System with Large $\rho$

### Average Cost

![Average Cost Graph](image)

### Approximately Optimal Test Interval

![Approximately Optimal Test Interval Graph](image)

**Observations:**

1. The optimal average costs increase with the dependence ($\beta$) between units.
2. The optimal test intervals decrease as the importance of the system ($C_u$) increases and decreases as the dependence between units increases, i.e., the system with larger $\beta$ has longer test interval.
Sensitivity Analysis—High Important System with Small $\rho$

**Average Cost**

**Approximately Optimal Test Interval**

**Observations:**

1. The optimal average costs increase with the dependence ($\beta$) between units.
2. The system with larger $\beta$ has longer test interval when the system is less important (small $C_u$), however, the optimal test intervals decrease as the importance increases. Moreover, the system with larger $\beta$ decreases more significantly.
Sensitivity Analysis—High Important System with Large $\rho$

**Average Cost**

![Average Cost Graph]

**Approximately Optimal Test Interval**

![Approximately Optimal Test Interval Graph]

**Observations:**

1. The optimal average costs increase with the dependence ($\beta$) between units.

2. The system with larger $\beta$ has longer test interval when the system is less important (small $C_u$), however, the optimal test intervals decrease as the importance increases. Moreover, the system with larger $\beta$ decreases more significantly.
Conclusions

- Uniformly staggered testing is recommended for identical units in parallel.

- Almost simultaneous testing is recommended for two-unit parallel systems with significantly different units. Unequal test interval is also suggested.

- If there exists CCF (on demand) between units,
  - the larger $\beta$ system has, the more cost is required,
  - the larger $\beta$ system has, the longer test interval is suggested for less important systems,
  - the larger $\beta$ system has, the shorter test interval is suggested for more important system.